Technical Notes

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Heat transfer in concentric annuli with moving cores—fully developed turbulent flow with arbitrarily prescribed heat flux

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1. INTRODUCTION

PROBLEMS involving fluid flow and heat transfer with the moving core of a solid body or liquid in an annular geometry can be found in many engineering practices, such as in manufacturing (e.g. extrusion, drawing and hot rolling), in transportation (e.g. trains travelling at high speed in a long tunnel or underground railways), in nuclear reactor operation (e.g. inverted annular film boiling) etc. In such processes, the fluid flow involved can be either laminar or turbulent and the moving body continuously exchanges heat with the surrounding environment.

In our previous studies, [1, 2], we presented the solutions on the problems of fully developed laminar and turbulent fluid flows and heat transfer in a concentric annulus with a moving core. For the laminar fluid flow [1], the solutions were obtained for the cases of one wall only heated with the other insulated and of the inner and outer tubes for any heat flux ratio. The solutions for the latter were obtained through the influence coefficients [3], which are evaluated from the fundamental solution from the definition.

For the case of turbulent fluid flow, the solution was presented for the condition of a constant heat flux at the inner core only with the outer wall insulated [2]. To complement this, the solutions are presented here, for the turbulent heat transfer in an annulus for the cases of the outer wall only heated with the inner wall insulated, and of the inner and outer walls for any heat flux ratio.

2. ANALYSIS

2.1. Case for the outer wall only heated and the inner insulated For the prediction of temperature distribution and heat transfer rates, a modified mixing length model for flow turbulence is used for the analysis [2]. The intermediate details of the mathematical development of the analysis can be easily deduced from ref. [2] and therefore are not presented here. Appropriate adjustments were made in the present analysis for the matching conditions to accommodate the different boundary condition from that of the case for the inner wall only heated and the outer insulated.

2.2. Cases for both walls heated independently

For the cases where both wall surfaces are heated independently, the Nusselt numbers on the two surfaces for any heat flux ratio may be calculated, utilizing the superposition method [1, 3]. This is because the governing energy equation for the present study is linear and homogeneous. The Nusselt numbers for asymmetric heating are then obtained through influence coefficients [4] given as:

$$Nu_{i} = \frac{Nu_{ii}}{1 - \theta_{i}^{*}(q_{Rk}/q_{Ri})}$$
(1)

where j = i then k = o and j = o then k = i. Here, q_{Ri} and q_{Ro} are defined as positive into the fluid.

The Nusselt numbers, Nu_i and Nu_o , are defined as:

$$Nu_j \equiv \frac{h_j 2(R_o - R_i)}{k}.$$
 (2)

The heat transfer coefficient is defined as:

$$h_i \equiv q_{\rm R_I}/(T_{\rm R_I} - T_{\rm b}) \tag{3}$$

where j = i or o. The influence coefficients, θ_i^* and θ_o^* , are defined as [3]:

$$\theta_i^* \equiv \frac{(T_{\rm b} - T_{\rm R_i})_{kk}}{(T_{\rm R_i} - T_{\rm b})_{ji}} \left[\frac{q_{\rm R_i} \cdot 2(R_{\rm o} - R_{\rm i})}{k} \right], \tag{4}$$

where for Case A : j = i and k = o and for Case B : j = o and k = i.

3. RESULTS AND DISCUSSION

The range of parameters considered are:

The radius ratio (α): 0.2, 0.5, 0.8 and 0.99

The relative velocity (U^*) : 0.0, 0.2, 0.4, 0.6, 0.8 and 1.0

The Reynolds numbers (Re): 10^4 , 3×10^4 , 10^5 and 10^6

Prandtl numbers (Pr): 0.72, 10 and 50.

The results are presented in Table 1 and in Figs. 1 and 2. The numerical values for the parameters in the table seem not to change monotonously with increasing values of U^* and this could be due to the combined effects of U^* , α , Pr and Re.

The results for $U^* = 0.0$ are almost identical to those of Kays and Crawford [4]. The insignificant difference is due to the different turbulence models used in the analyses.

The predicted Nusselt numbers for the cases of one wall only heated with the other insulated, Nu_{ij} , for the range of the relative velocity, U^* , between 0.0 and 1.0 are plotted against the radius ratio, α , in Fig. 1 for Reynolds and Prandtl numbers of 10⁵ and 0.72, respectively. The effect of the relative velocity is seen to decrease with a decreasing value of α as was the case of the effect on the friction factor as seen previously in ref. [2]. It was also observed that the effect of the relative velocity on heat transfer is the opposite of that of the friction factor; i.e. the heat transfer increases with an increasing value of the relative velocity.

NOMENCLATURE					
h k Nu	heat transfer coefficient thermal conductivity Nusselt number	θ^*	influence coefficients.		
Pr	Prandtl number	Subscripts			
R	radius	ь	bulk		
Re	Reynolds number	i	inner		
Т и	temperature fluid velocity in x-direction	ii	constant heat rate at the inner wall with the outer insulated		
U_{-}	core velocity	j,k	i or o		
U^*	dimensionless relative velocity, $U/u_{\rm b}$.	0	outer		
ireek symbols		00	constant heat rate at the outer wall with the inner insulated		
α	radius ratio, R_i/R_o	R	radius.		



FIG. 1. Nusselt numbers.

;			1						
oped turbulent flow in concentric annuli with moving cores; $0 \leq U^* \leq 1$	10*	$\theta^*_{\rm o}$	0.0470 0.0467	0.0463 0.0455 0.0442	0.0416 0.0155 0.0160 0.0164 0.0168 0.0170 0.0170	0.0050 0.0055 0.0059 0.0063 0.0065 0.0065	$\begin{array}{c} 0.1061\\ 0.1047\\ 0.1026\\ 0.0993\\ 0.0944\\ 0.0866 \end{array}$	$\begin{array}{c} 0.0378\\ 0.0380\\ 0.0381\\ 0.0378\\ 0.0378\\ 0.0376\\ 0.0345\end{array}$	0.0154 0.0157 0.0159 0.0157 0.0150 0.0150
		ΰ*	0.3101 0.3137	0.3193 0.3288 0.3464	0.3874 0.1101 0.1080 0.1063 0.1054 0.1075 0.1075	0.0468 0.0461 0.0455 0.0454 0.0454 0.0462	0.2305 0.2313 0.2334 0.2334 0.2381 0.2492 0.2831	$\begin{array}{c} 0.0842 \\ 0.0817 \\ 0.0794 \\ 0.0777 \\ 0.0786 \\ 0.0786 \\ 0.0919 \end{array}$	0.0365 0.0351 0.0336 0.0324 0.0324 0.0322
		Nu_{oo}	1037 1041	1047 1056 1070	1095 5414 5478 5558 5559 5786 5943	11976 11918 11823 11672 11422 11422	996.5 998.1 1002 1010 1024 1052	5205 5248 5318 5422 5573 5573	11 924 11 885 11 819 11 690 11 437 10 883
		Nu_{ii}	1442 1523	1626 1768 1981	2383 7295 7662 7943 8428 9767	16478 16863 17253 17578 1758	1118 1184 1272 1396 1592 1997	5840 5989 6190 6486 7017 8525	13487 13771 14091 14415 1461 15412
	3×10^4 10 ⁴	θ^*_{o}	0.0632 0.0623	0.0610 0.0592 0.0569	cccou 0.0189 0.0189 0.0184 0.0178 0.0157 0.0157	0.0072 0.0071 0.0069 0.0067 0.0063 0.0053	0.1348 0.1331 0.1331 0.1261 0.1200 0.1108	0.0427 0.0419 0.0407 0.0390 0.0336 0.0332	0.0161 0.0158 0.0154 0.0154 0.0138 0.0138
		0 [‡] *	0.3806 0.3797	0.3796 0.3808 0.3848	0.3974 0.1158 0.1141 0.1125 0.1113 0.1108 0.1133	0.0429 0.0420 0.0412 0.0401 0.0401 0.0407	0.2827 0.2804 0.27786 0.2778 0.2778 0.2793	0.0909 0.0889 0.0868 0.0851 0.0840 0.0861	0.0343 0.0333 0.0334 0.0334 0.0334 0.0315 0.0315 0.0313 0.0313
		Nu_{∞}	163.2 163.9	164.9 166.2 168.1	1/0.8 667.9 665.5 665.4 666.3 668.9	1217 1212 1208 1205 1204	155.8 156.0 156.7 157.9 157.9 153.2	663.2 658.4 654.5 651.7 651.7 651.9	1222 1210 1199 1183 1183
		$Nu_{\rm ii}$	224.1 233.7	245.7 261.6 284.3	223.3 920.3 948.2 984.2 1108 1108 1250	1675 1717 1773 1852 1973 2214	173.3 181.2 191.6 205.9 227.5 267.9	750.5 773.2 804.5 850.0 923.9 1080	1389 1422 1470 1542 1664
		θ*	0.0719 0.0712	0.0702 0.0686 0.0664	0.0651 0.0210 0.0208 0.0198 0.0198 0.0179	0.0080 0.0079 0.0078 0.0076 0.0074 0.0070	0.1510 0.1501 0.1481 0.1481 0.1446 0.1390 0.1301	0.0460 0.0455 0.0447 0.0432 0.0411 0.0378	0.0176 0.0175 0.0173 0.0168 0.0160 0.0148
fully deve		0 ⁱ *	0.4215 0.4177	0.4143 0.4115 0.4104	0.4148 0.1224 0.1203 0.1183 0.1165 0.1155 0.1155	$\begin{array}{c} 0.0456\\ 0.0447\\ 0.0438\\ 0.0438\\ 0.0424\\ 0.0426\\ \end{array}$	0.3123 0.3079 0.3036 0.2998 0.2974 0.3004	$\begin{array}{c} 0.0965\\ 0.0941\\ 0.0917\\ 0.0895\\ 0.0879\\ 0.0887\end{array}$	$\begin{array}{c} 0.0368\\ 0.0357\\ 0.0347\\ 0.0336\\ 0.0336\\ 0.0328\\ 0.0329\end{array}$
befficients, 1		$Nu_{\alpha\alpha}$	65.47 65.73	66.10 66.63 67.35	68.41 240.9 240.4 240.1 240.1 240.2 240.2 240.9	429.5 427.3 425.5 423.0 423.0 423.0	62.33 62.37 62.59 63.04 63.81 63.05	240.4 238.2 236.4 234.2 234.2	432.2 427.1 422.5 418.5 418.5 413.4
l influence e		$Nu_{\rm ii}$	89.37 92.44	96.24 101.2 108.1	119.4 332.6 341.6 353.1 368.9 392.5 435.2	590.9 604.7 622.9 648.5 687.6 761.2	69.05 71.63 74.97 79.50 86.21 98.13	271.7 278.8 28.8 303.3 326.7 373.9	490.8 501.3 516.6 540.6 579.0 661.6
Table I. Nusselt numbers and	104	$\theta^*_{\circ \theta}$	0.0818 0.0817	0.0812 0.0802 0.0785	0.0241 0.0241 0.0241 0.0237 0.0237 0.0237 0.0231	0.0097 0.0098 0.0098 0.0097 0.0093	0.1684 0.1691 0.1687 0.1687 0.1668 0.1627 0.1548	0.0518 0.0521 0.0520 0.0512 0.0496 0.0465	0.0209 0.0213 0.0215 0.0214 0.0209 0.0198
		0*	0.4650 0.4589	0.4527 0.4469 0.4420	0.4404 0.1347 0.1323 0.1323 0.1279 0.1262 0.1262	0.0523 0.0513 0.0503 0.0493 0.0485 0.0482	0.3431 0.3371 0.3309 0.3197 0.3197	0.1067 0.1040 0.1014 0.0988 0.0966 0.0958	0.0426 0.0415 0.0403 0.0392 0.0381 0.0375
		$N u_{\alpha \alpha}$	30.40 30.47	30.59 30.78 31.05	92.144 99.160 98.74 98.31 98.31 98.35	172.5 171.4 170.3 169.4 168.7 168.7	28.91 28.91 28.94 29.06 29.31 29.31	99.30 98.17 97.17 96.33 95.70 95.37	174.0 171.6 169.3 167.3 167.3 165.5 164.1
-		Nu_{ii}	41.41 42.46	43.76 45.43 47.74	0.42 1.40.3 140.3 144.3 144.3 149.8 157.9 171.8	237.5 242.4 248.8 257.8 271.3 295.3	32.00 32.91 34.08 35.66 37.97 42.03	112.2 114.6 118.0 123.0 131.0 146.4	197.6 201.1 206.4 214.4 2214.8 254.8
, KARPAL ANNUAR	R_{ℓ}	Pŗ.		0.72	10	50	0.72	10	50
and the subscreen	*/1	د	$\begin{array}{c} x = 2 \\ 0.0 \\ 0.2 \end{array}$	0.4 0.6 0.8	1.0 0.2 0.4 0.8 0.8 1.0	$\begin{array}{c} 0.0\\ 0.2\\ 0.6\\ 0.8\\ 1.0\end{array}$	x = 0.5 0.0 0.4 0.8 0.8 0.8 1.0	0.0 0.2 0.6 1.0	0.0 0.4 0.8 1.0

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0.1474 0.1452 0.1452 0.1416 0.1362 0.1381 0.1281 0.1156	0.0534 0.0533 0.0530 0.0530 0.0521 0.0503 0.0461	0.0228 0.0230 0.0229 0.0223 0.0223 0.0209 0.0175	0.1676 0.1649 0.1605 0.1539 0.1539 0.1441 0.1287	0.0610 0.0607 0.0601 0.0588 0.0588 0.0565	0.0264 0.0265 0.0263 0.0255 0.0255 0.0236
0.1886 0.1884 0.1890 0.1916 0.1996 0.2304	0.0689 0.0663 0.0638 0.0619 0.0622 0.0750	0.0298 0.0283 0.0267 0.0252 0.0246 0.0246	0.1695 0.1688 0.1688 0.1706 0.1773 0.2070	0.0617 0.0591 0.0567 0.0546 0.0546 0.0546	0.0267 0.0251 0.0235 0.0219 0.0212 0.0280
985.0 984.9 987.4 994.2 1008 1040	5147 5173 5229 5328 5488 5728	11913 11873 11816 11705 11455 10836	984.7 983.6 985.4 991.7 1006 1039	5139 5156 5205 5301 5466 5725	11 927 11 885 11 830 11 830 11 480 10 824
1020 1080 1161 1279 1472 1902	5336 5467 5653 5940 6477 8104	12 378 12 599 12 862 13 152 13 415 14 499	986.2 1044 1122 1238 1238 1431 1878	5148 5270 5446 5726 6260 7956	11 948 12 140 12 373 12 641 12 901 14 153
0.1837 0.1815 0.1776 0.1776 0.1714 0.1620 0.1474	0.0593 0.0581 0.0563 0.0537 0.0499 0.0445	0.0244 0.0220 0.0213 0.0203 0.0189 0.0168	0.2075 0.2050 0.2004 0.1931 0.1819 0.1641	0.0673 0.0659 0.0638 0.0607 0.0562 0.0495	0.0255 0.0250 0.0242 0.0230 0.0230 0.0213 0.0187
0.2326 0.2298 0.2273 0.2257 0.2257 0.2257	0.0735 0.0733 0.0712 0.0692 0.0678 0.0694	0.0286 0.0276 0.0266 0.0257 0.0249 0.0253	0.2097 0.2068 0.2041 0.2020 0.2016 0.2016	0.0680 0.0659 0.0638 0.0618 0.0618 0.0616	0.0258 0.0248 0.0238 0.0238 0.0229 0.0221
153.7 153.7 154.1 154.1 155.2 157.3 161.0	665.9 658.8 652.7 648.1 645.6 645.6	1233 1216 1216 1201 1187 1177 1177	153.7 153.5 153.8 154.8 154.8 157.0 161.0	669.4 661.1 653.9 648.4 645.2 645.8	1242 1222 1204 1189 1177 1177
158.7 165.9 175.6 189.3 210.9 254.2	692.1 711.7 739.6 782.2 854.7 1022	1284 1312 1353 1420 1539 1828	153.9 160.9 170.2 183.8 205.4 250.7	670.5 688.5 714.8 755.8 827.4 1001	1244 1268 1307 1370 1487 1787
0.2044 0.2035 0.2007 0.1955 0.1870 0.1730	0.0634 0.0628 0.0615 0.0594 0.0560 0.0507	$\begin{array}{c} 0.0243\\ 0.0242\\ 0.0239\\ 0.0232\\ 0.0232\\ 0.0219\\ 0.0199\end{array}$	0.2303 0.2293 0.2261 0.2200 0.2098 0.1926	0.0718 0.0712 0.0697 0.0631 0.0630 0.0565	0.0276 0.0275 0.0271 0.0271 0.0271 0.0271 0.0222
0.2575 0.2530 0.2848 0.2440 0.2420 0.2429	0.0804 0.0779 0.0754 0.0731 0.0712 0.0717	0.0308 0.0297 0.0287 0.0287 0.02876 0.0267 0.0267	0.2327 0.2281 0.2235 0.2190 0.2155 0.2171	0.0726 0.0702 0.0677 0.0654 0.0634 0.0637	0.0278 0.0268 0.0258 0.0258 0.0238 0.0237
61.55 61.43 61.53 61.92 62.70 64.12	241.8 238.6 235.9 233.8 232.3 232.0	436.9 429.8 423.4 417.7 413.1 410.1	61.59 61.39 61.44 61.79 62.60 64.12	243.3 239.7 236.5 234.0 232.3 231.8	440.3 432.3 424.9 418.4 413.1 409.6
63.44 65.79 68.92 73.31 80.06 92.95	251.2 257.1 265.9 279.4 302.4	454.8 463.0 476.2 497.6 535.7 624.3	61.67 63.93 66.97 71.30 78.09 91.66	243.7 249.1 257.3 270.2 293.0 345.9	441.1 448.3 460.3 480.6 518.0 610.3
0.2261 0.2276 0.2272 0.2244 0.2182 0.2054	0.0708 0.0714 0.0713 0.0701 0.0675 0.0625	0.0286 0.0293 0.0296 0.0295 0.0287 0.0268	0.2540 0.2557 0.2554 0.2554 0.2522 0.2446 0.2289	0.0800 0.0808 0.0806 0.0792 0.0761 0.0698	0.0322 0.0331 0.0335 0.0334 0.0325 0.0300
0.2835 0.2776 0.2715 0.2655 0.2597 0.2570	0.0892 0.0866 0.0839 0.0813 0.0777 0.0777	0.0359 0.0348 0.0337 0.0337 0.0314 0.0314	0.2566 0.2509 0.2449 0.2388 0.2332 0.2399	0.0808 0.0782 0.0756 0.0730 0.0706 0.0693	0.0326 0.0315 0.0304 0.0293 0.0282 0.0274
28.65 28.52 28.52 28.55 28.55 29.30 29.30	100.1 98.51 97.09 95.87 94.93	176.3 173.0 169.9 164.6 164.6	28.72 28.54 28.52 28.52 28.77 29.31	100.8 99.05 97.43 96.02 94.92 94.27	177.9 174.2 170.7 167.4 164.6 162.4
29.50 30.35 31.46 33.01 33.01 39.78	104.0 105.9 108.9 113.5 121.4 138.0	183.5 186.2 190.5 197.9 210.9 239.8	28.75 29.57 30.65 32.18 34.57 39.19	101.0 102.8 105.5 109.9 117.7 135.0	178.3 180.5 184.4 191.3 204.1 234.1
8 0.72	10	50	99 0.72	10	50
$\alpha = 0.8$ 0.0 0.2 0.4 0.6 0.8 1.0	0.0 0.2 0.6 0.6 1.0	0.0 0.2 0.6 0.8 1.0	$\begin{array}{l} \alpha = 0.5 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.8 \\ 0.8 \\ 1.0 \end{array}$	0.0 0.2 0.6 0.6 1.0	0.0 0.2 0.4 0.6 0.8 1.0



FIG. 2. Influence coefficients.

Figure 1 also illustrates that while the effect of the relative velocity on Nu_{ii} (the Nusselt number for the case of the inner wall only heated and the outer insulated) is significant, the effect on Nu_{oo} (the case of the outer wall heated and the inner insulated) is negligibly small. The same trend can be observed for the effect of the radius ratio on heat transfer. The analytical study of Kays and Crawford [4] of concentric annuli with stationary cores (i.e. $U^* = 0$) is compared with that of the present analysis. Despite the different method of analysis employed, it was seen that the agreement is very good for the range of the parameters studied. No comparison was made for the case of concentric annuli with moving cores (i.e. $U^* > 0$) with other works as there is none available in the open literature.

From Table 1, it can be seen that the Nusselt numbers increase with increasing values of Prandtl number but its combined effect with that of the relative velocity on the Nusselt number are similar to those observed in Fig. 1. The effects of α and U^* on the influence coefficients, θ_i^* and θ_{α}^* , are shown in Fig. 2 for Reynolds and Prandtl numbers of 10⁵ and 0.72, respectively.

4. CONCLUDING REMARKS

A complete solution for the fully developed turbulent flow and heat transfer in concentric annuli with moving cores has been made.

The study showed that for equal conditions, increasing relative velocity were observed for the following changes:

(i) a decrease in friction factor; and

(ii) an increase in Nusselt number.

However, the effect of the relative velocity seems to diminish with decreasing value of the radius ratio and especially for the case of the outer wall heated and the inner wall insulated.

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Study of heat transfer from buried nuclear waste canisters

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Conduction limit

INTRODUCTION

IT HAS been proposed that radioactive waste from nuclear power plants be disposed of in cylindrical containers by burying them under the surface of the earth. To carry out safety analysis and to gauge the impact of this proposal on the environment it is necessary to determine flow patterns and heat transfer rates in the vicinity of these containers. The present work gives a summary of analytical and numerical results for temperature distribution in and around a canister buried in a saturated porous medium. Heating of the canister surface takes place because of the decay of radioactive waste contained within it. It is important to know the maximum and minimum temperatures on the cylinder surface since they decide the magnitude of the transport coefficients and the extent of the thermal stresses. Heat transfer from the cylinder to its surroundings will occur due to one of the following mechanisms: conduction, buoyancy-driven convection of the pore fluid and forced convection due to natural ground water movement. Solutions for these problems are available when the surface of the cylinder has a prescribed temperature. Results have been presented here for a single and an array of cylinders with specified heat flux on their surface.

FORMULATION

Fluid flow in a saturated homogeneous isotropic porous medium is taken to be governed by Darcy's law,

$$\mathbf{u} = -K(\nabla p + \rho g \mathbf{k})/\mu \tag{1}$$

the incompressibility constraint $\nabla \cdot \mathbf{u} = 0$ and the energy transport equation,

$$T_{t} + \mathbf{u} \cdot \nabla T = \alpha \nabla^{2} T.$$
 (2)

In the absence of buoyancy effects ρ is a constant and $\nabla \cdot \mathbf{u} = \nabla \cdot K \nabla \rho = 0$. For flow past a single cylinder buried in a uniform medium, K is a constant and $\nabla^2 \rho = 0$. Using potential theory the velocity components can be determined as u - iv = U. $(1 - R^2/z^2)$ where z = x + iy and $i = \sqrt{-1}$. For an array of canisters we solve the equation $\nabla \cdot K \nabla \rho = 0$ numerically by assigning a small value for K over the cylinders and unit value in the flow region. Equation (2) has been solved here subject to the constant heat flux condition, $-T_r$ (r = R) = q. The results are presented in dimensionless form using R as the length scale, R^2/α as the time scale, the approach velocity U as the velocity scale and qR as the temperature scale. In free convection problems the velocity scale is α/R . Convection problems are assumed to have reached steady state since they occur in boundary layers.

RESULTS

In the absence of a super-imposed flow the conduction problem follows the dimensionless equation,

$$T_t = \nabla^2 T$$

subject to T(t = 0) = 0. This equation can be solved by Fourier transforms. For an isothermal boundary condition T(r = 1) = 1 the solution for the wall heat flux is

$$-T_{r}|_{r=1} = \left(\frac{2}{\pi}\right)^{2} \int_{0}^{\infty} \frac{1}{\beta N(\beta)} \exp\left(-\beta^{2} t\right) d\beta$$

where $N(\beta) = J_0^2(\beta) + Y_0^2(\beta)$. This integral is evaluated numerically by Simpson's rule. The conduction solution also describes the local heat flux for steady forced flow parallel to the axis of a cylinder with *t* replaced by *z*/*Pe*. The latter problem has been solved in [1] using boundary-layer analysis. The two solutions are compared in Table 1.

For a heat flux boundary condition $(-T_r(r=1) = 1)$ we solve for the wall temperature as,

$$T(1,t) = \int_0^\infty \frac{R^2(\beta)}{\beta N(\beta)} [1 - \exp(-\beta^2 t)] d\beta$$

where $N(\beta) = J_1^2(\beta) + Y_1^2(\beta)$ and $R(\beta) = J_0(\beta)Y_1(\beta) - J_1(\beta)Y_0(\beta)$. The value of T attained by an isolated canister can increase further if more canisters are present in its neighbourhood. Consider a symmetric array of five canisters, four of which are placed on a square edge d and the fifth is placed at the centre. The temperature of the central canister is obtained by the principle of linear superposition. Calculations show that the minimum temperature is within 98% of the maximum temperature in Table 2.

Free convection [2]

The boundary-layer form of equations governing buoyancy-driven flow and heat transfer are given below.

Table 1. Comparison of heat flux values on an isothermal cylinder

t, z/Pe	Present	[1]
0.5	2.081	2.06
1	1.649	1.597
5	1.071	1.107